P-E-R-I-O-D-I-C

AND

APERIODIC

TESSELLATION

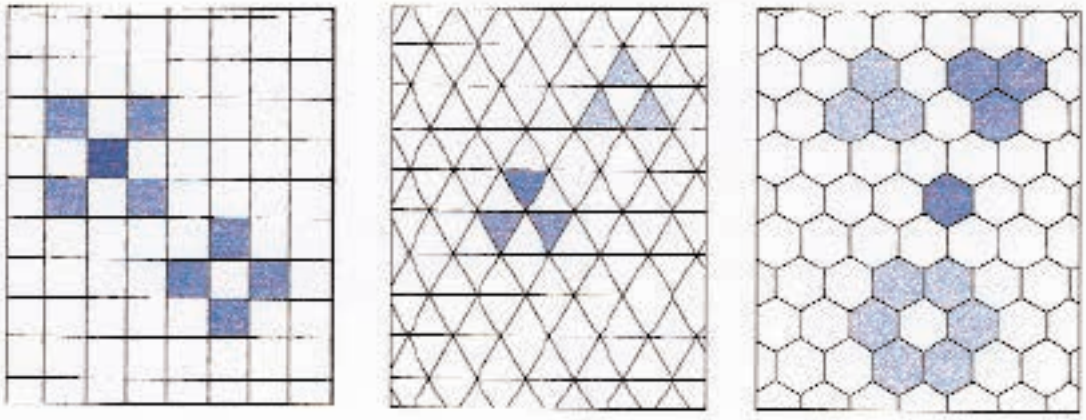
- BASIC CONCEPTS -

© 2014

ANTONIO PETICOV

When you fit individual tiles together with no gaps or overlaps to fill a flat space like a ceiling, wall, or floor, you have a **tiling** or **tessellation**.

You can imagine that you can use a variety of shapes to do this.

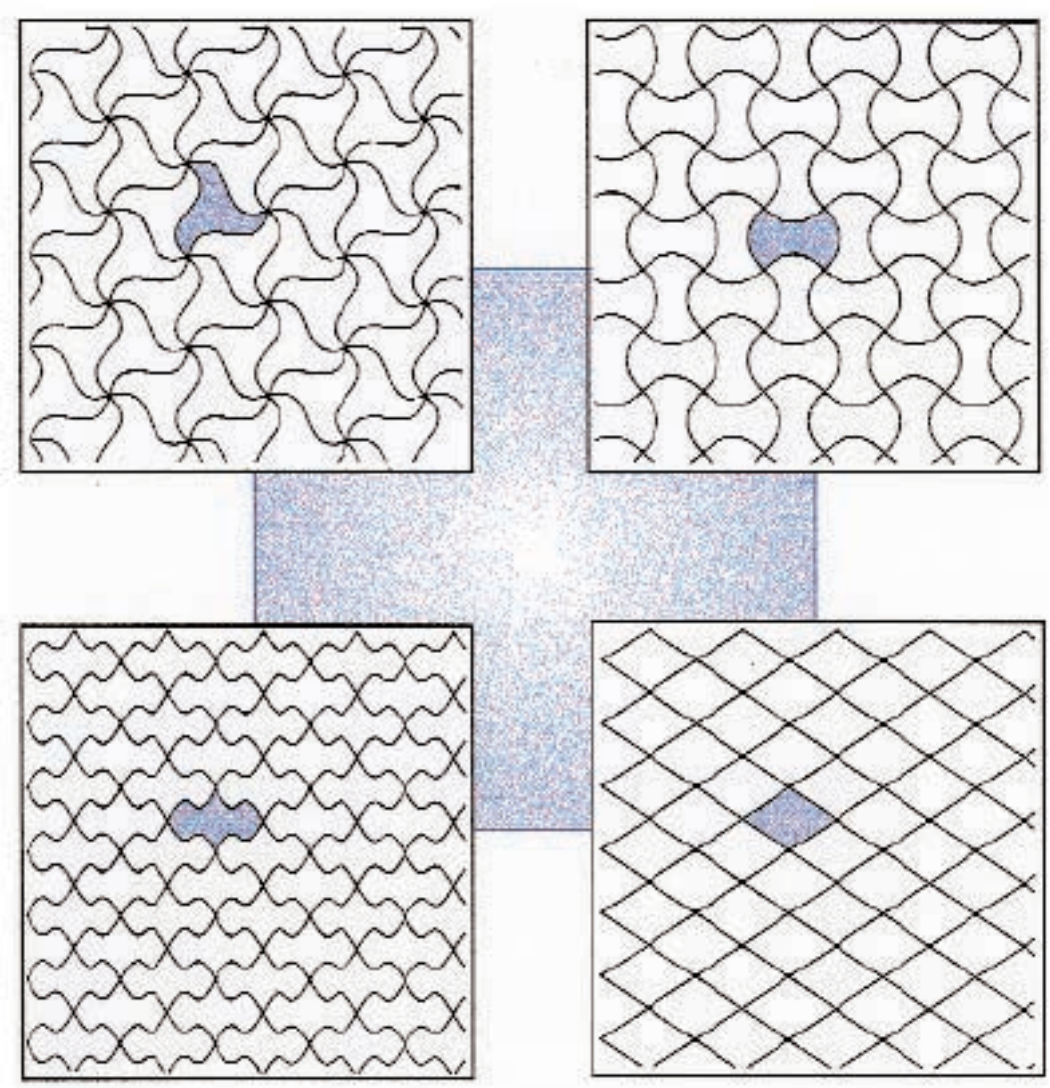


The majority of the TILINGS
that we know are made with
square, triangular or hexagonal
shapes,

but never like a regular pentagon.

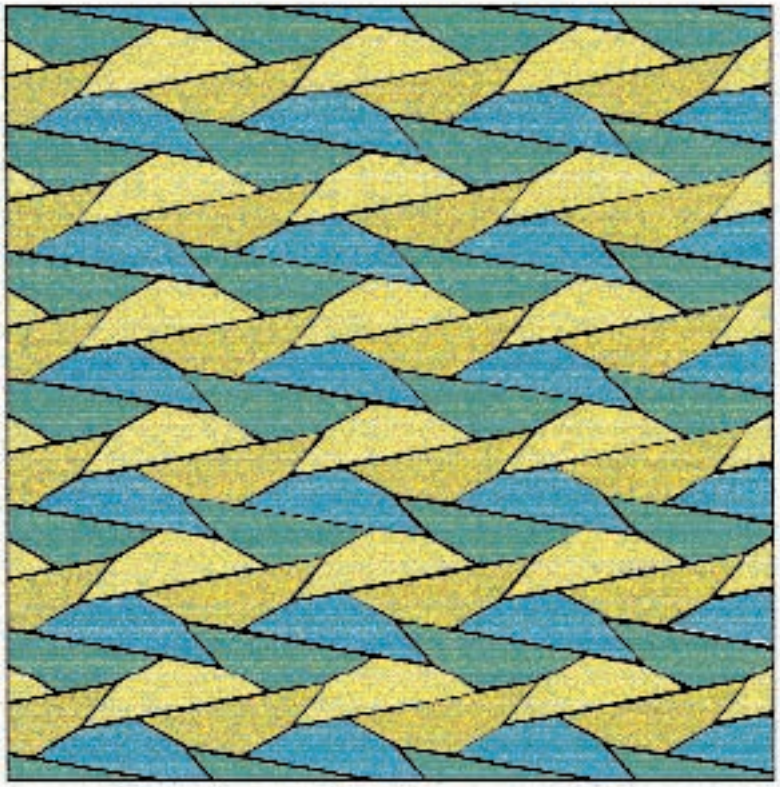
PERIODICAL TILING

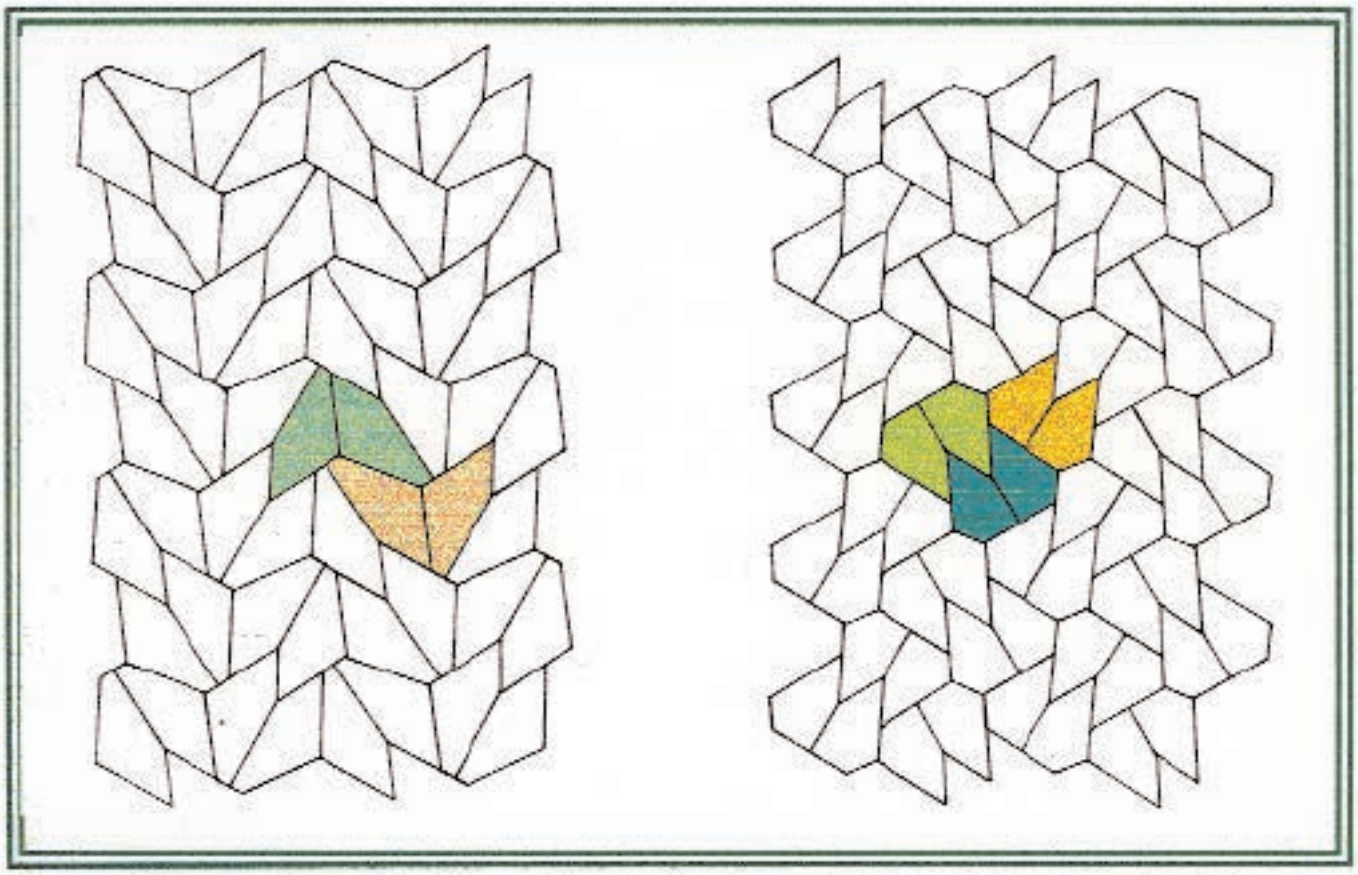
We call **Periodic Tiling** when
the design formed by its tiles
is found **repeated**
in other regions
inside the area that it covers

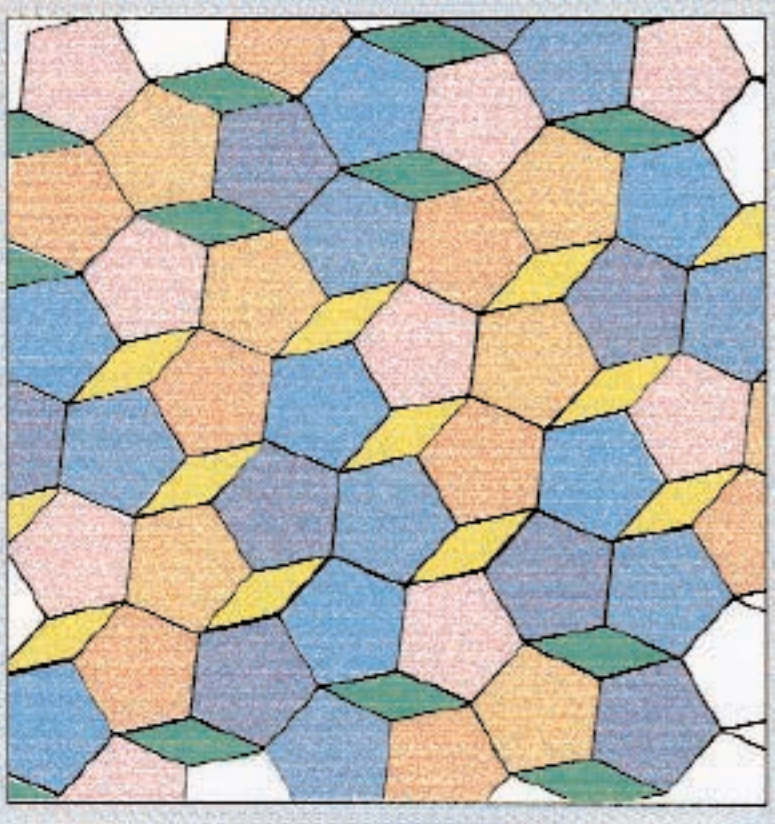


There are many other shapes
that we can use
to "tile" a plane **periodically**
using a great variety of
regular polyhedra...

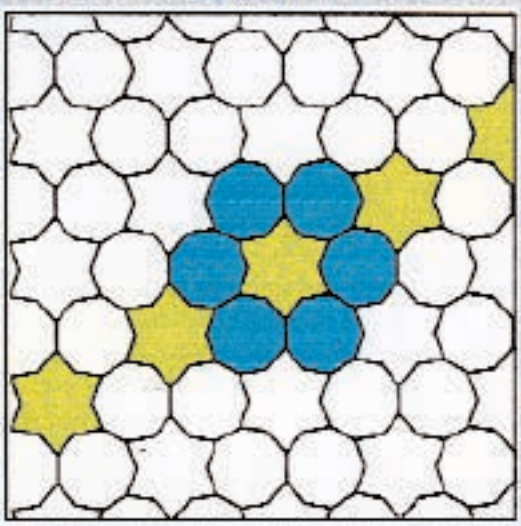
. . . AND **IRREGULAR**

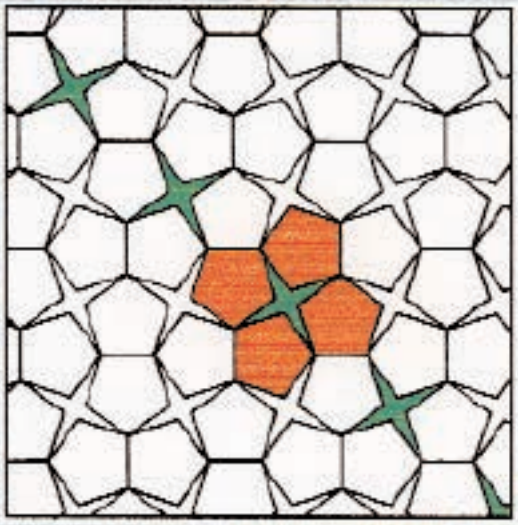






In the construction of each of the three examples of Periodic Tiling presented in this page two different regular polygons were used.





The repetition of the tiles in a tessellation can vary according to its rotation, translation and/or reflection.

ROTATION

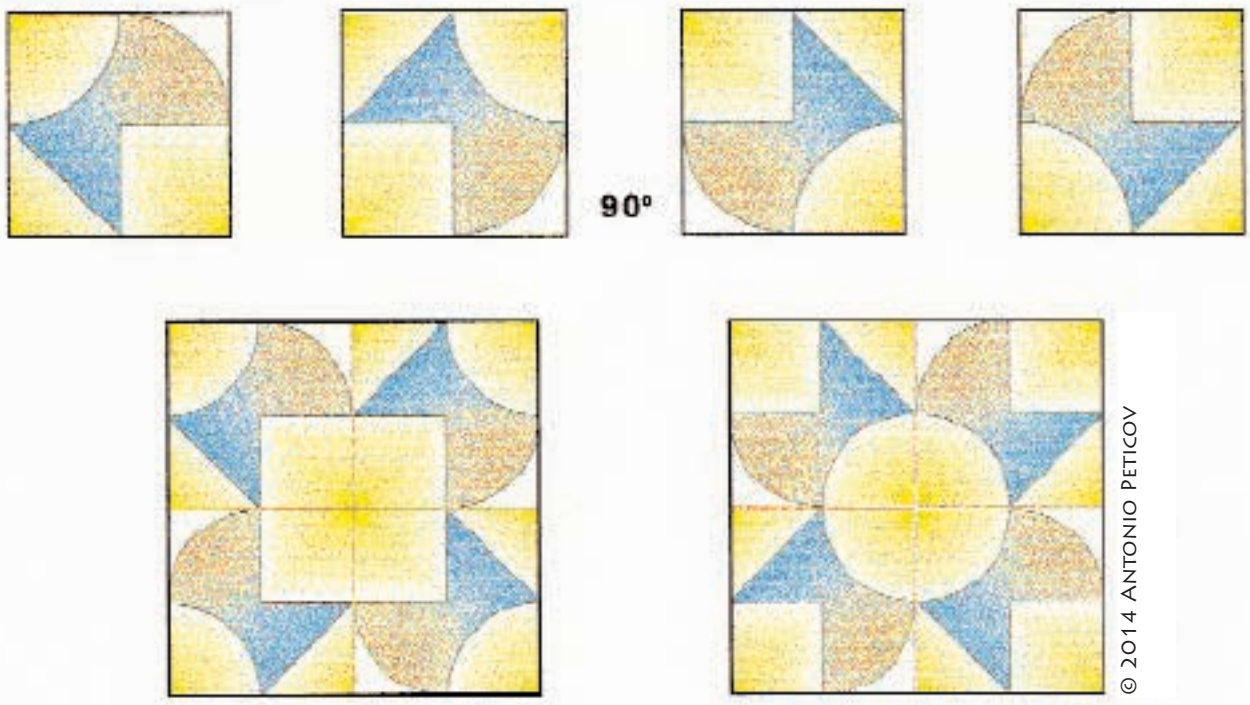
ROTATION

ROTATION

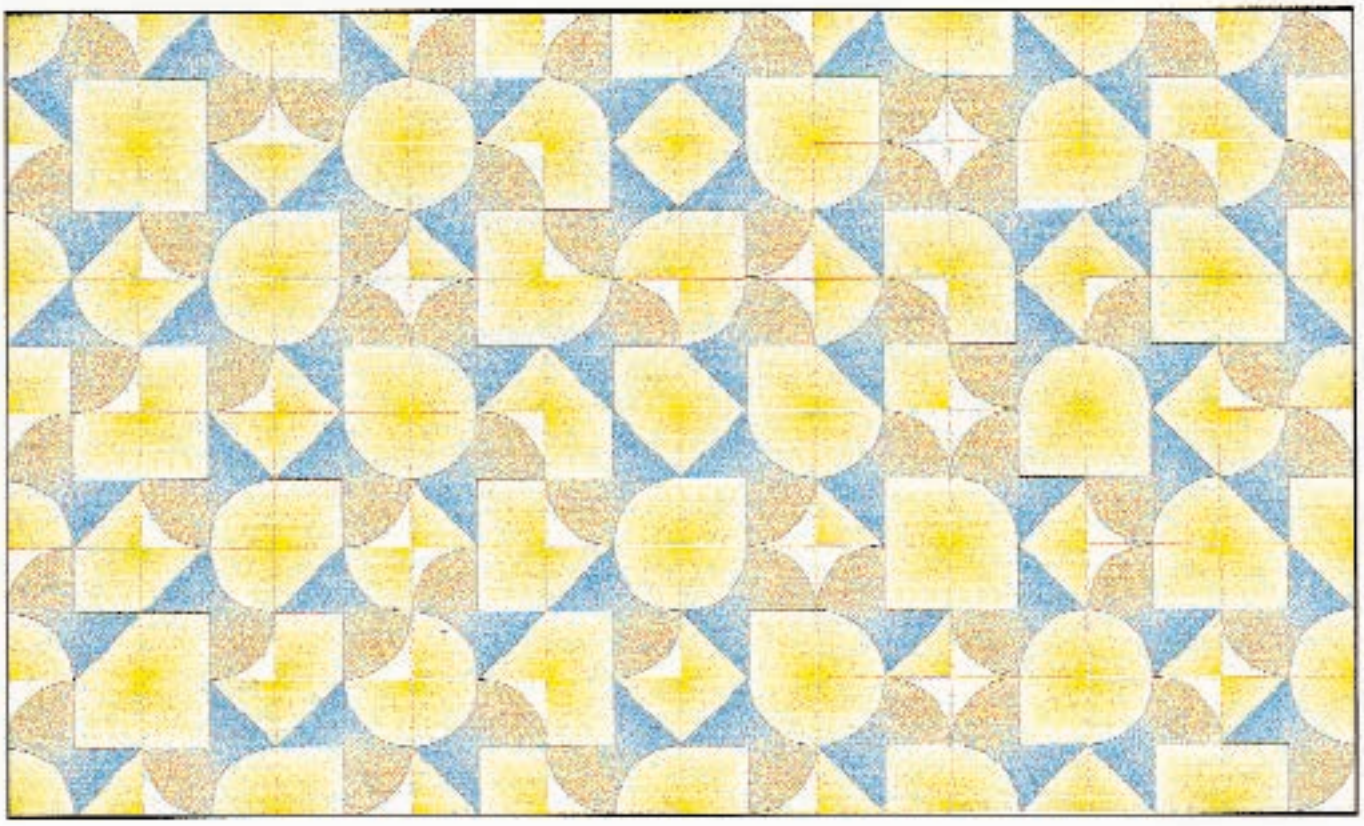
A tile suffers **rotation** when it is turned on its axis. Each rotation has a center and a angle.

The example of rotation presented above is a detail of the mural **"Anthropophagic Moment with Oswald de Andrade"** installed at the Republica station of the São Paulo metrô, in Brazil.

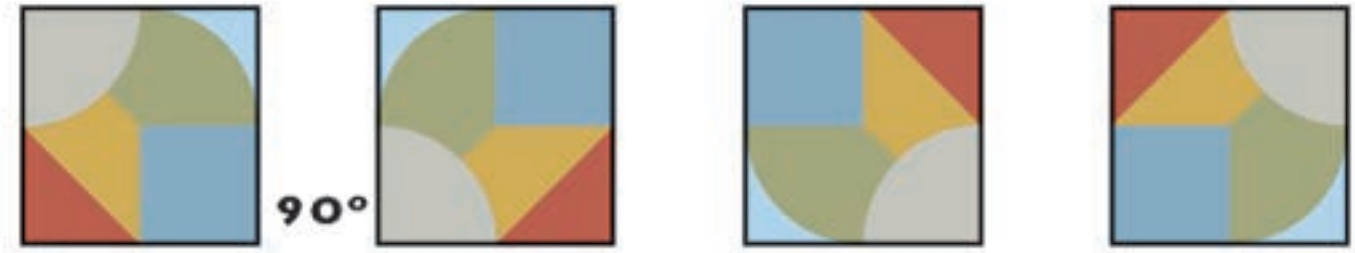
© 2014 Antonio Peticov



The **90°** rotation of this square tile enables the formation of a great number of geometric combinations.

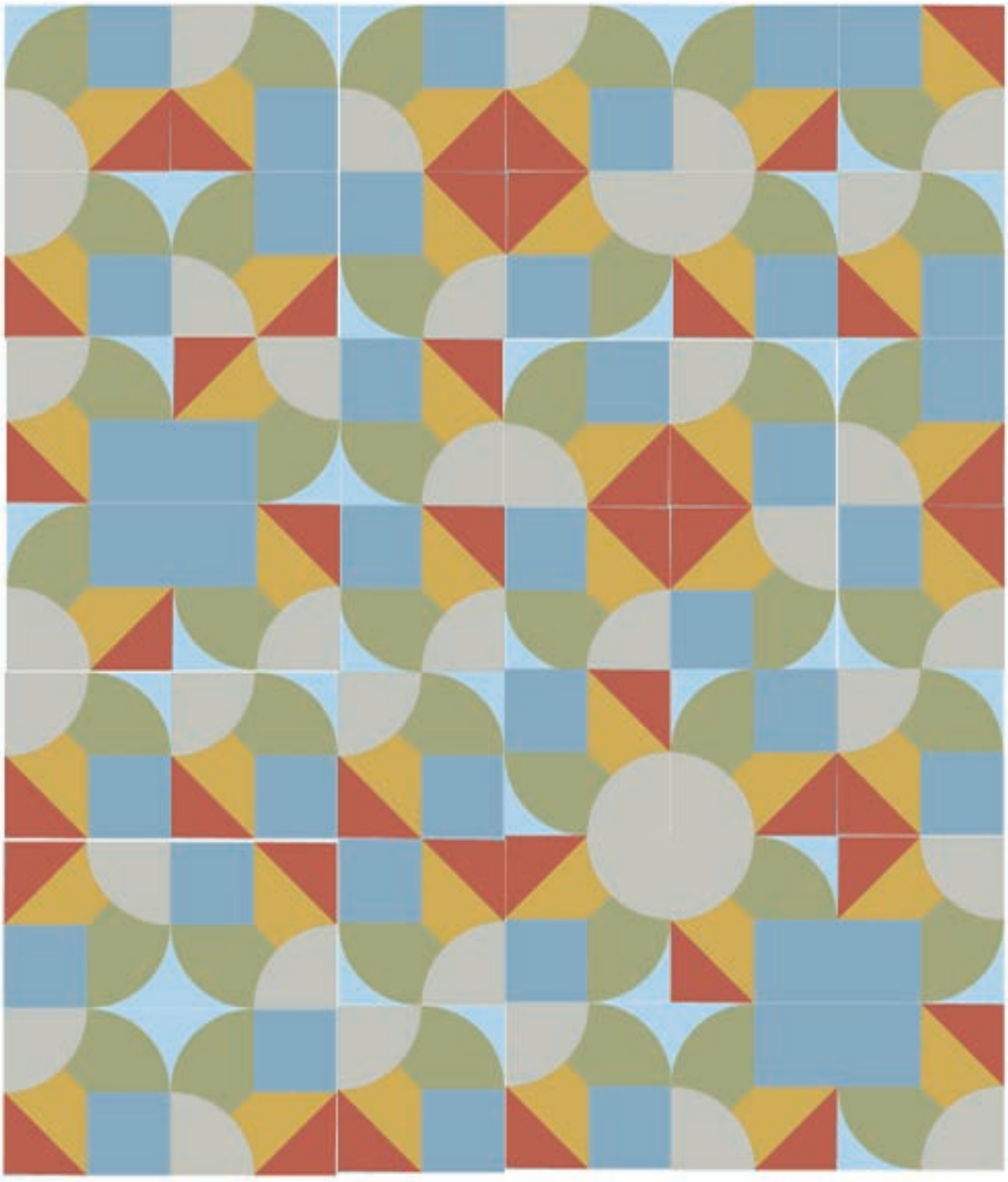


© 2014 Antonio Peticov



HYDRAULIC TILE

With **90°** turns and some combinations

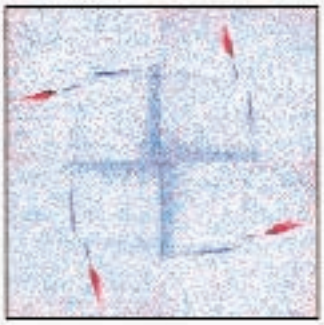


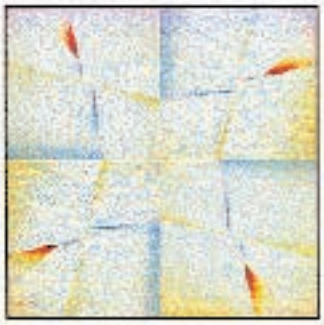
Lorenzo Borletti's apartment in São Paulo

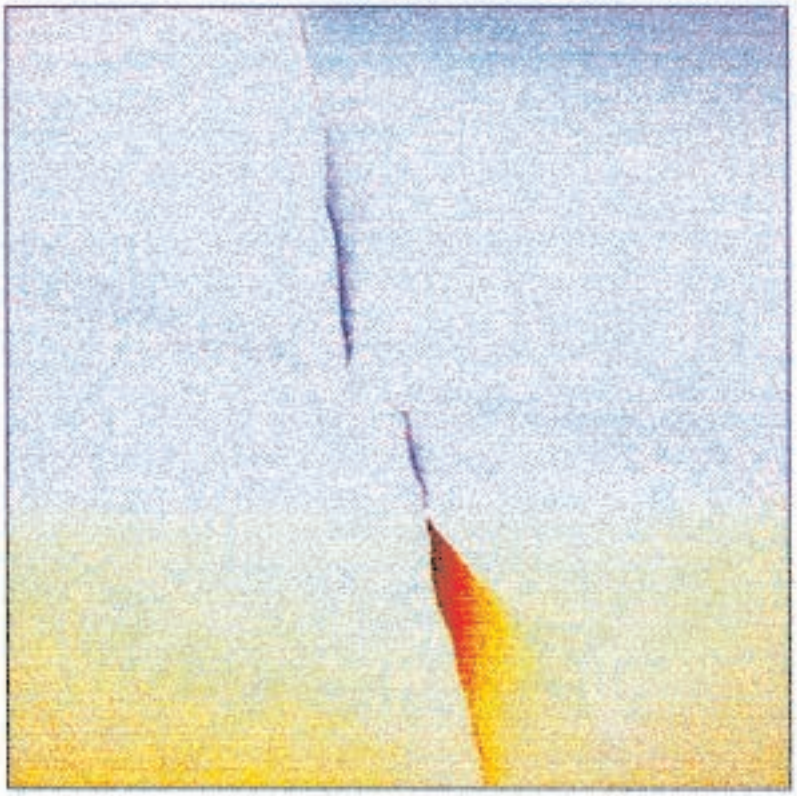
Lorenzo Borletti's apartment in São Paulo

“Saint Gerome’s Meditation” - acrylic on canvas - *180 x 140 cm* - ©2015 **Antonio Peticov**

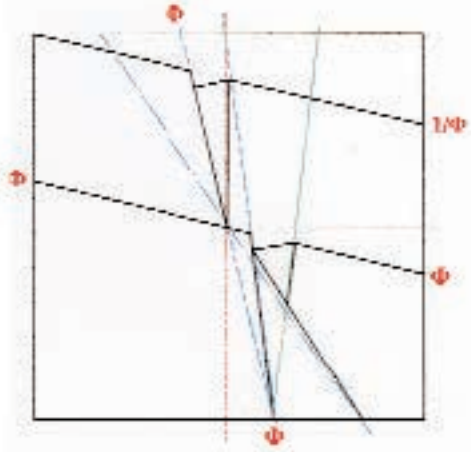
“A Espreita” - acrylic on canvas - *160 x 120 cm* - ©2015 **Antonio Peticov**







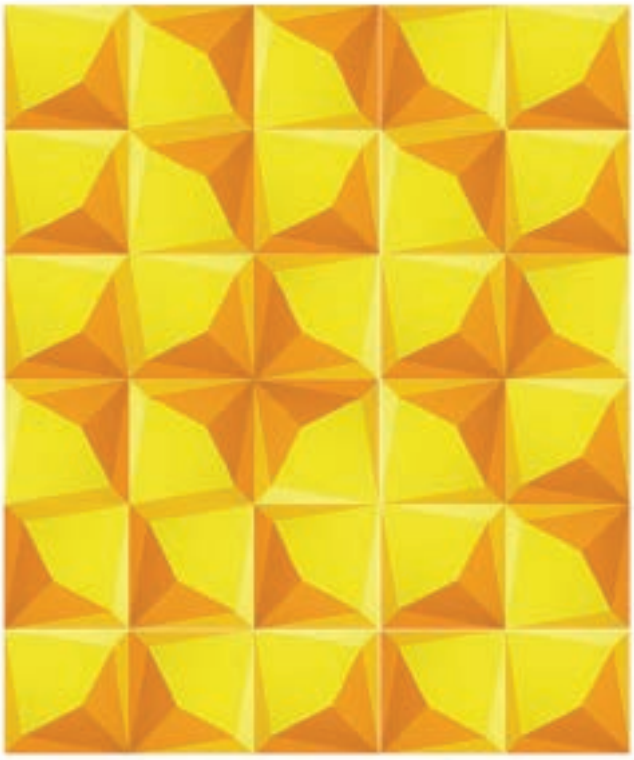
"DREAMING" *61,8 x 61,8 cm* © 2014 Antonio Peticov



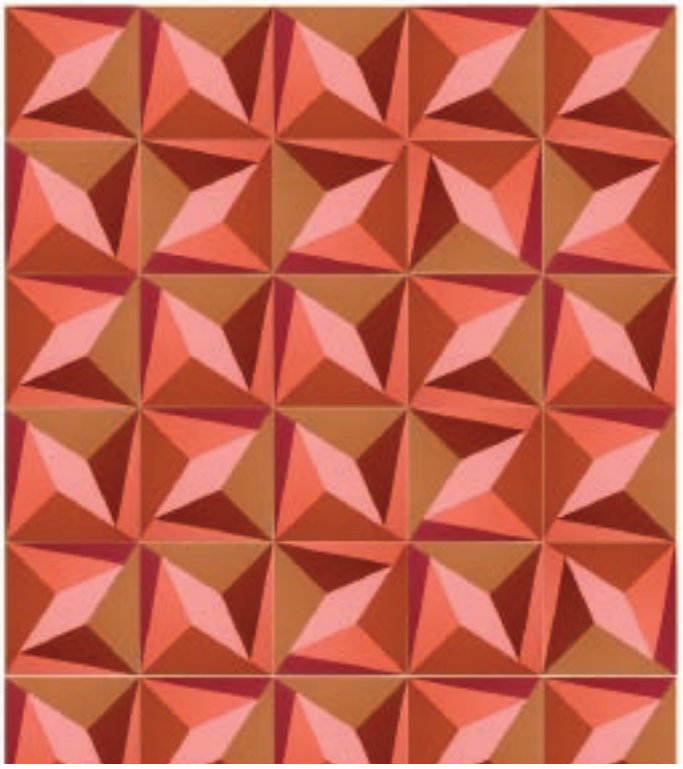
© 2014 Antonio Peticov

Amarelo

Ladrilho de porcelana com lado de 20 cm



Geomatrix 1



Geomatrix 2

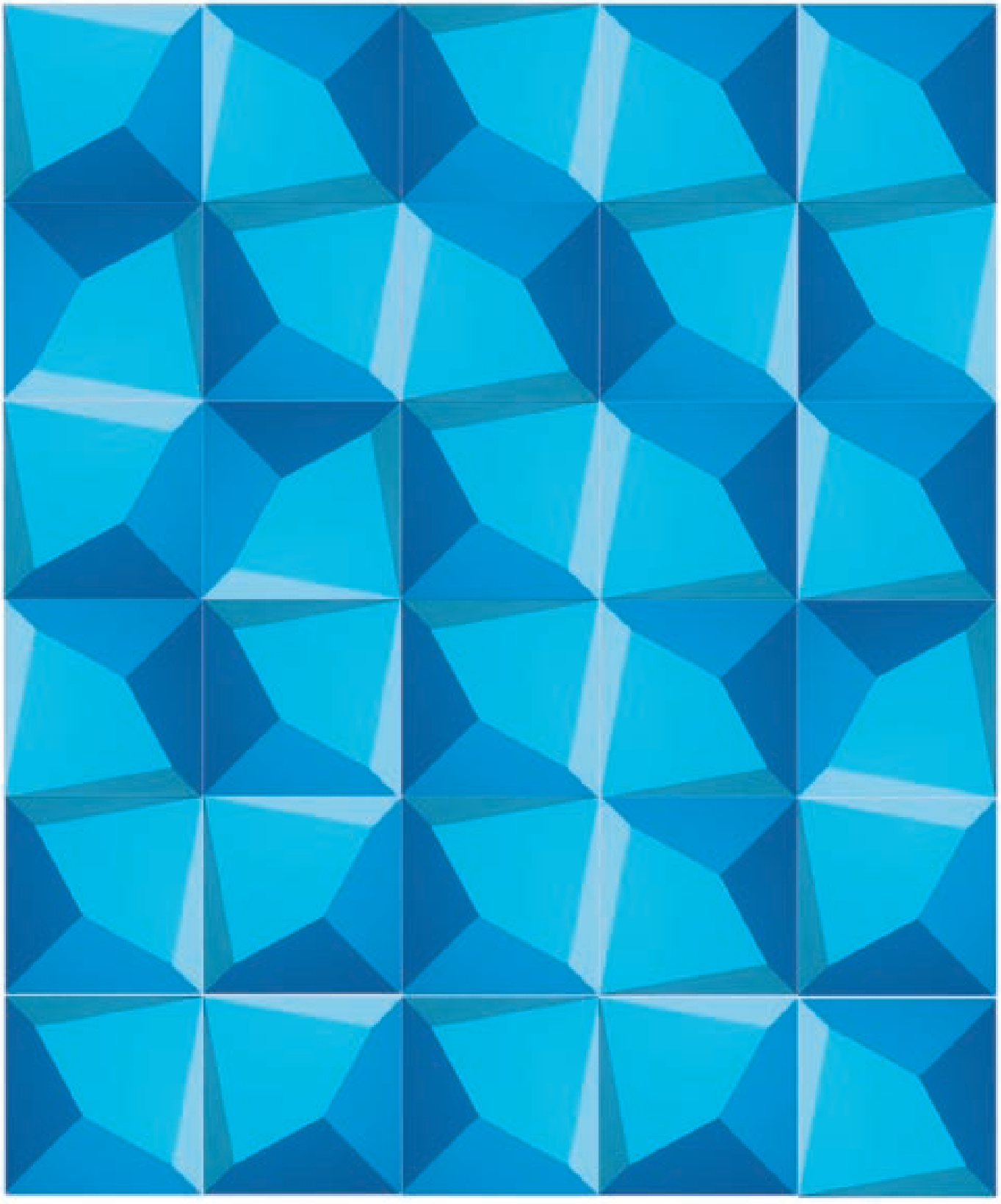
Ladrilho de porcelana com lado de 20 cm

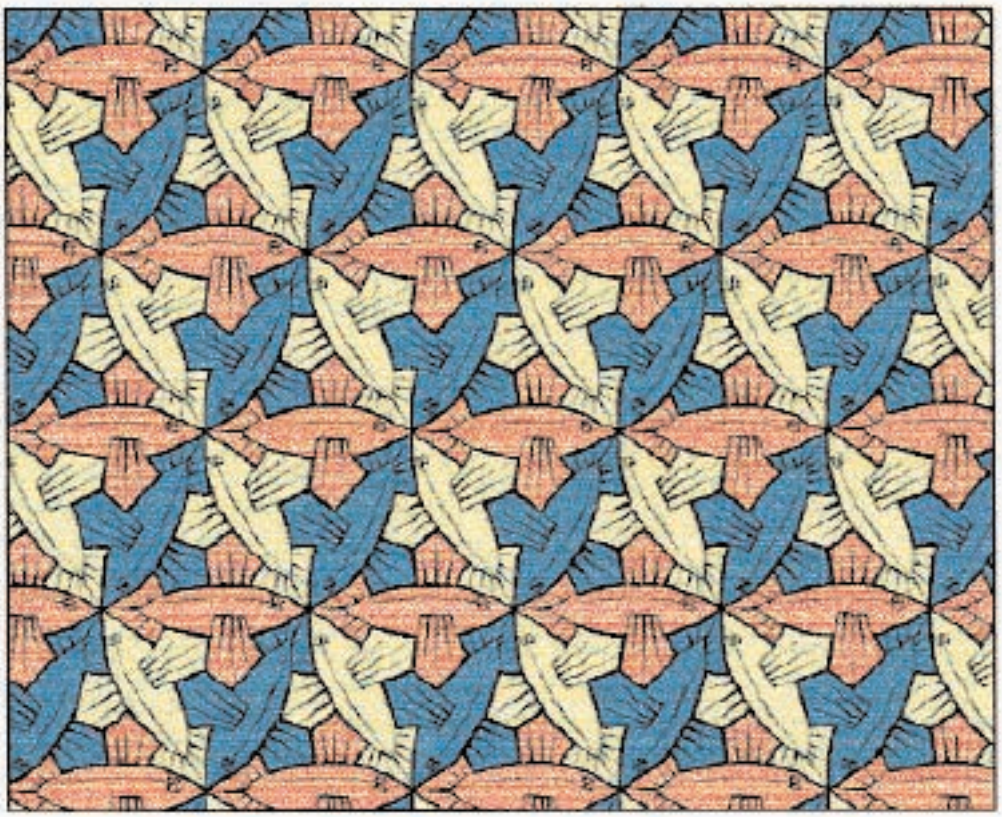
Cinza

Ladrilho de porcelana com lado de 20 cm

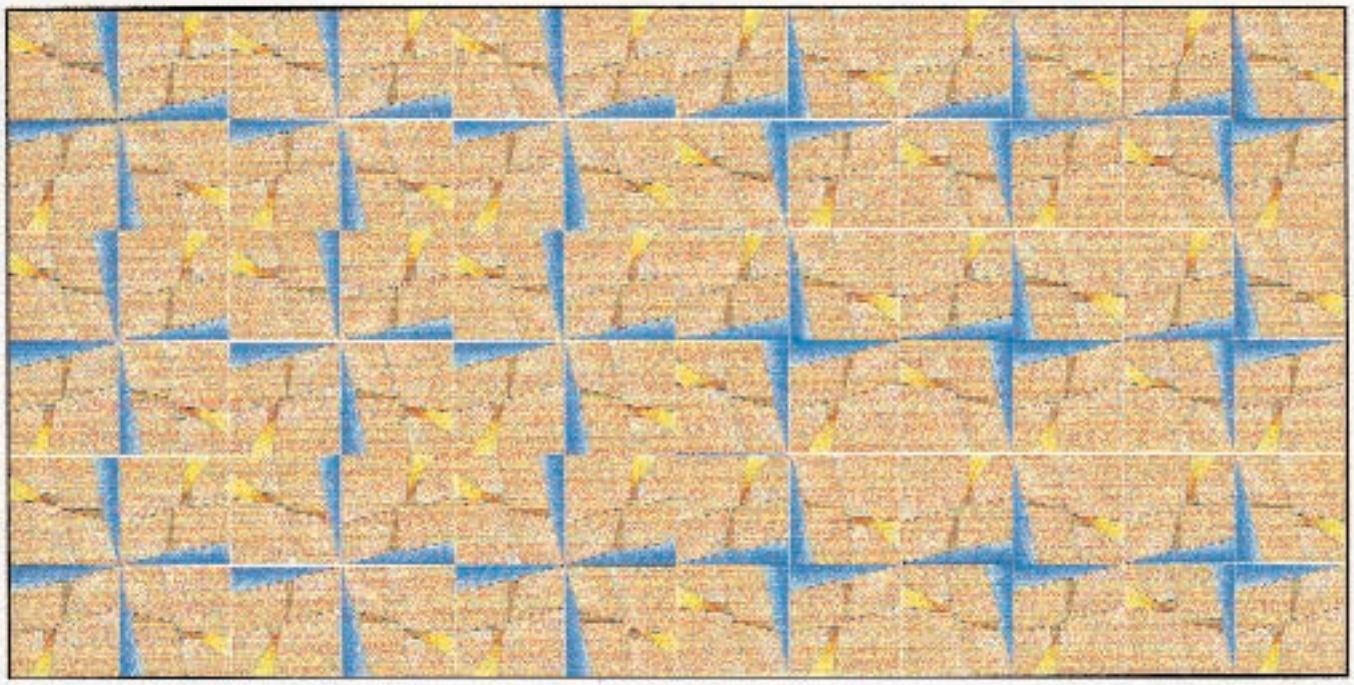
BLUE

Porcelaine Tile with 20 cm side





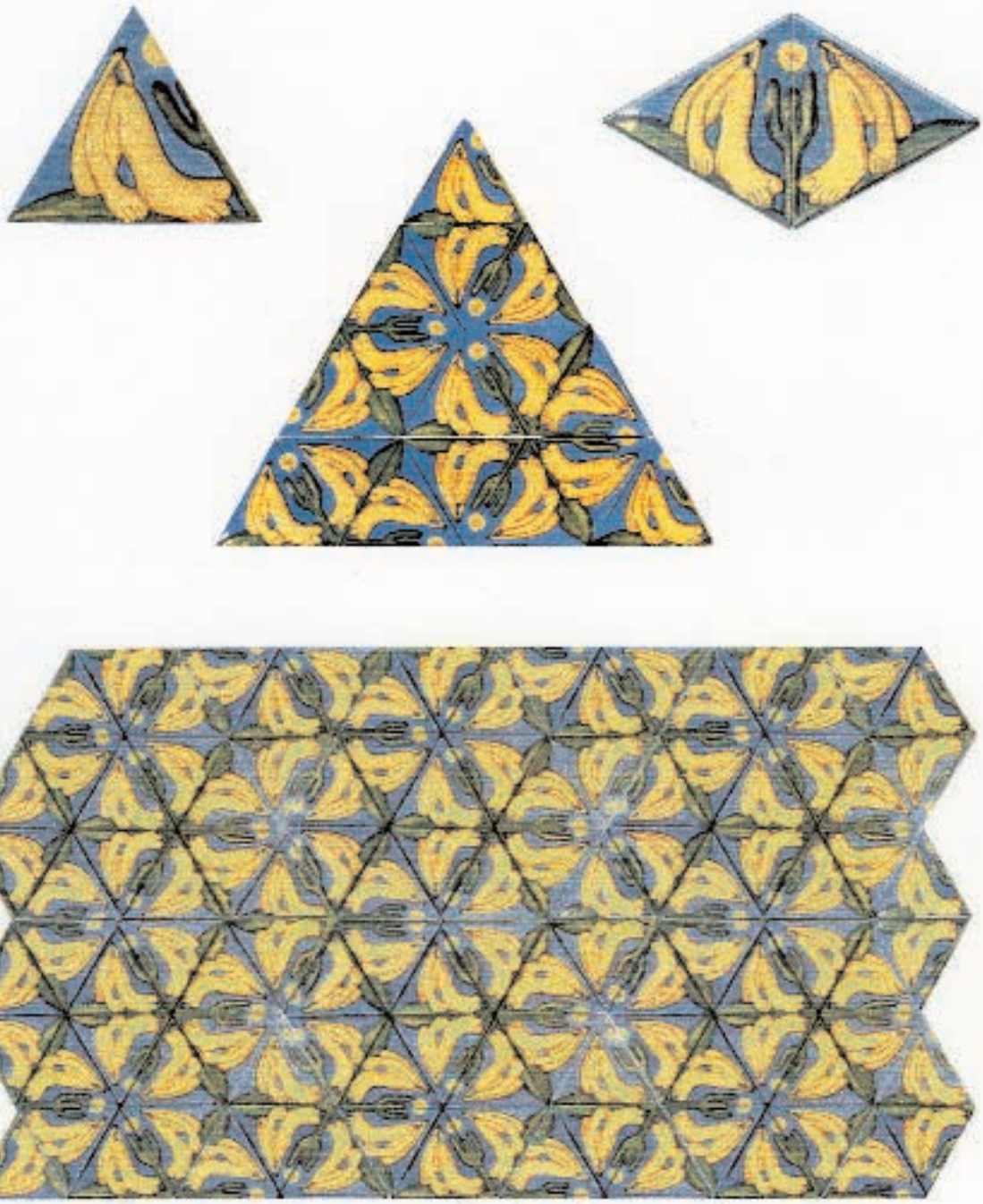
"Flying Fish"
With a **120°** rotation



© 2014 Antonio Peticov

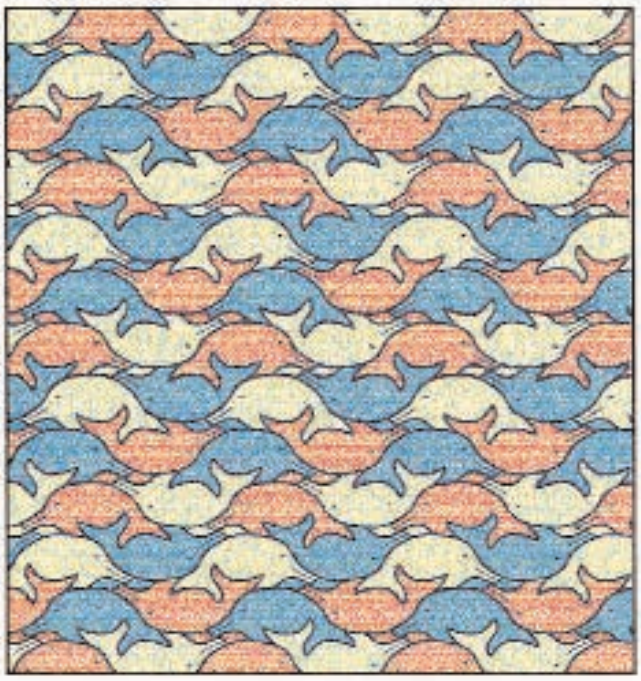
REFLECTION

THE REFLECTION OF A TILE CREATES ITS
MIRROR IMAGE.
EACH REFLECTION HAS ITS MIRROR LINE.



THE EXAMPLE OF REFLECTION PRESENTED ABOVE IS A DETAIL OF THE MURAL
"ANTHROPOPHAGIC MOMENT WITH OSWALD DE ANDRADE"
INSTALLED AT THE REPUBLICA STATION OF THE SÃO PAULO METRÔ, IN 1990.

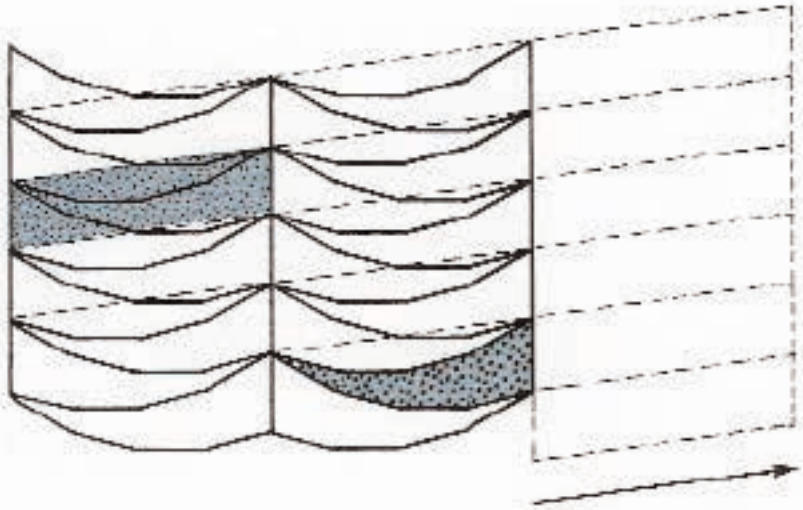
© 2014 ANTONIO PETICOV



Examples of **reflection**

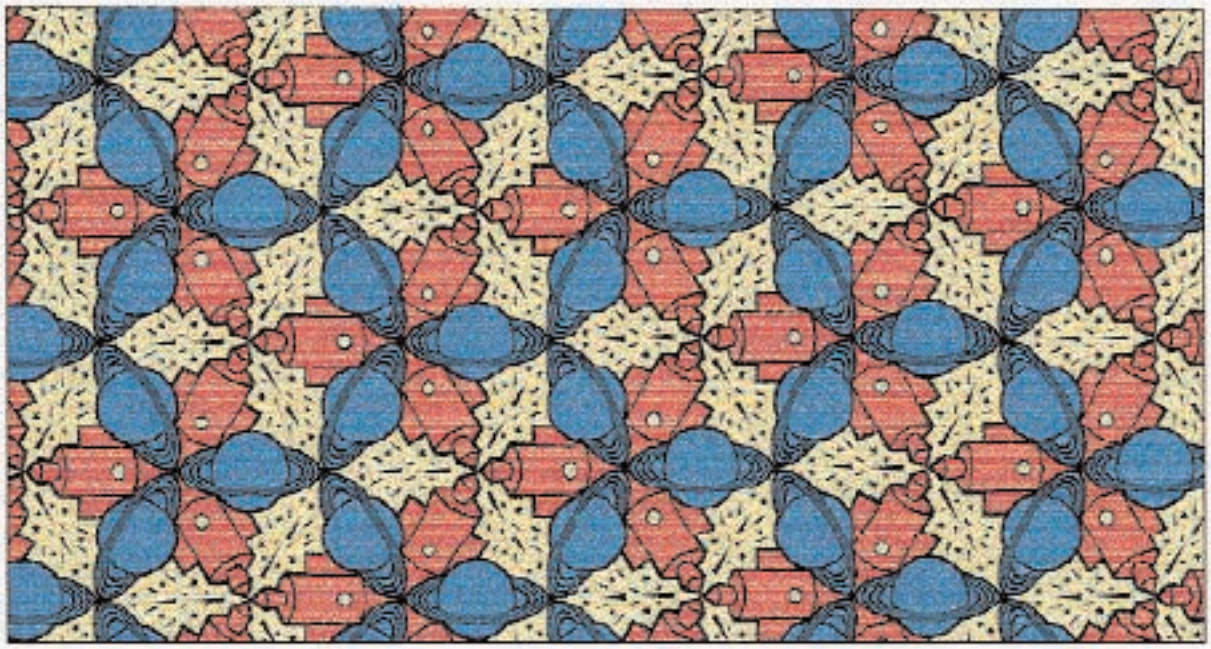
TRANSLATION

THE TRANSLATION OF A TILE OCCURS WHEN IT IS MOVED WITHOUT REFLECTION OR ROTATION. THE TRANSLATION HAS DIRECTION AND DISTANCE.

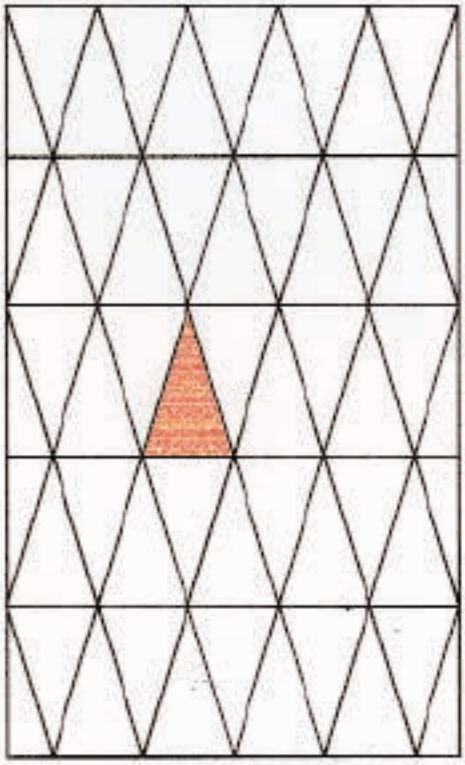


© 1988 M.C. Escher Heirs / Cordon Art – Baarn – Holland

M. C. ESCHER
CREATED THIS "TWO BIRDS"
THAT COMPOSE THIS PERIODIC TILING
IN 1945



ROTATION AND TRANSLATION
OF THREE TILES.



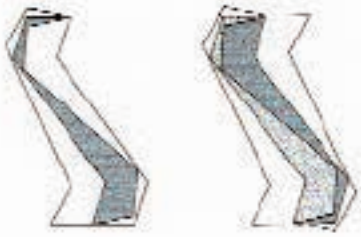
TYPICAL PERIODIC TESSELLATION
WITH ISOSELES TRIANGLES

NON-PERIODIC TESSELLATION
WITH THE SAME
ISOSELES TRIANGLES

"VERSATILE" POLIGONS

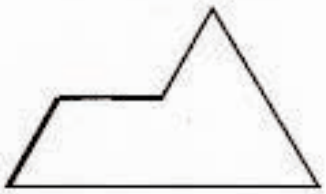
WE CALL A POLIGON "**VERSATILE**" WHEN IT ENABLES A TESSELLATION BOTH PERIODIC AND NON-PERIODIC

A **PERIODIC** TESSELLATION

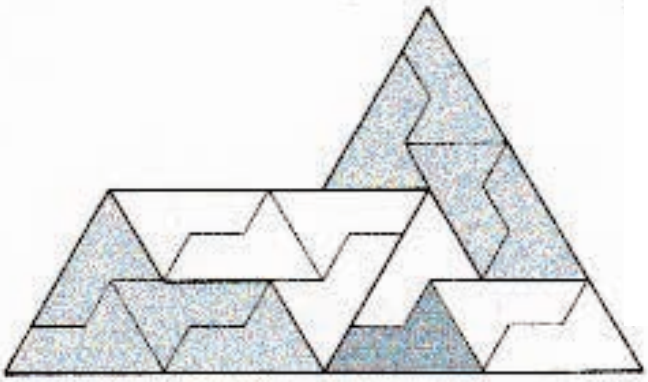


NOTE THAT THIS POLYGON COMES WITH THE POSSIBILITY TO BE COMPLETELY "EMBRACED" BY OTHER TWO PIECES ONLY, SIMILAR IN SIZE TO THE ORIGINAL

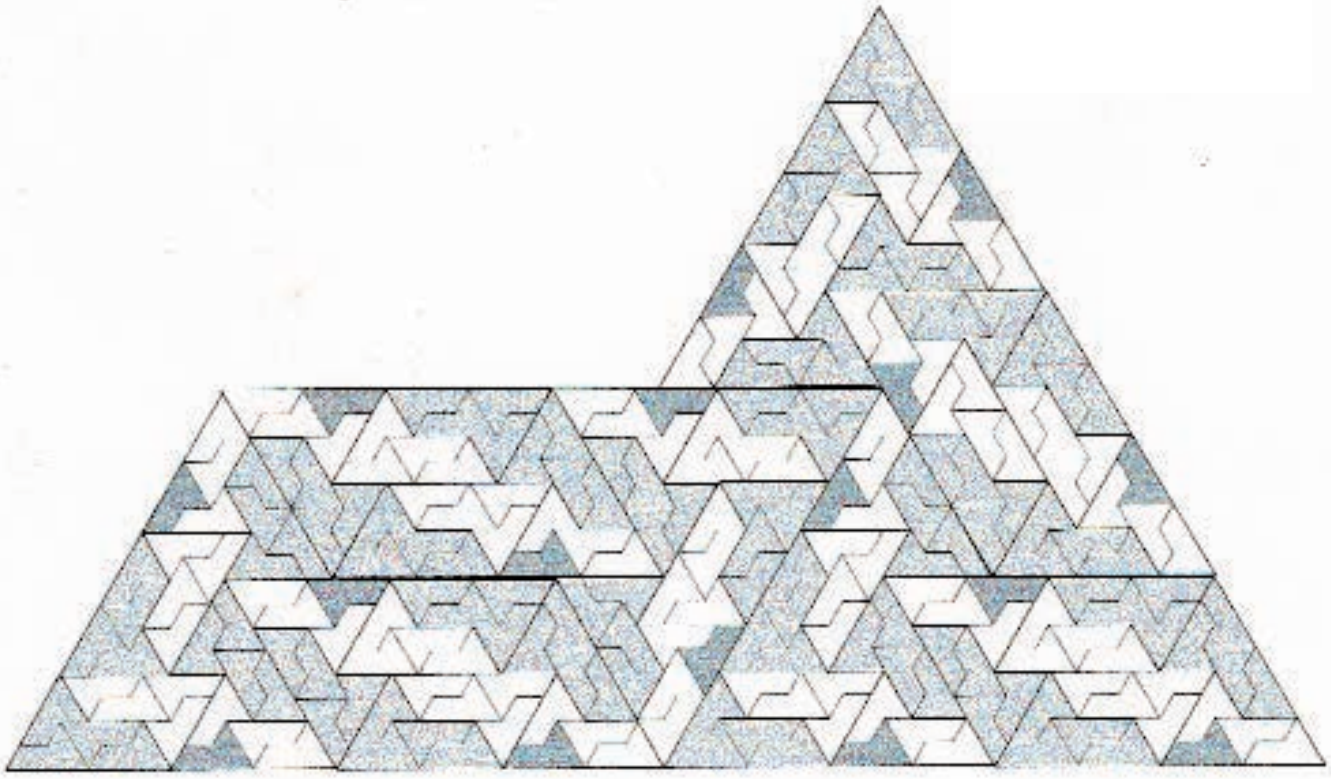
A **NON-PERIODIC** TESSELLATION
BY HEINZ VODERBERG



"REPTILES"

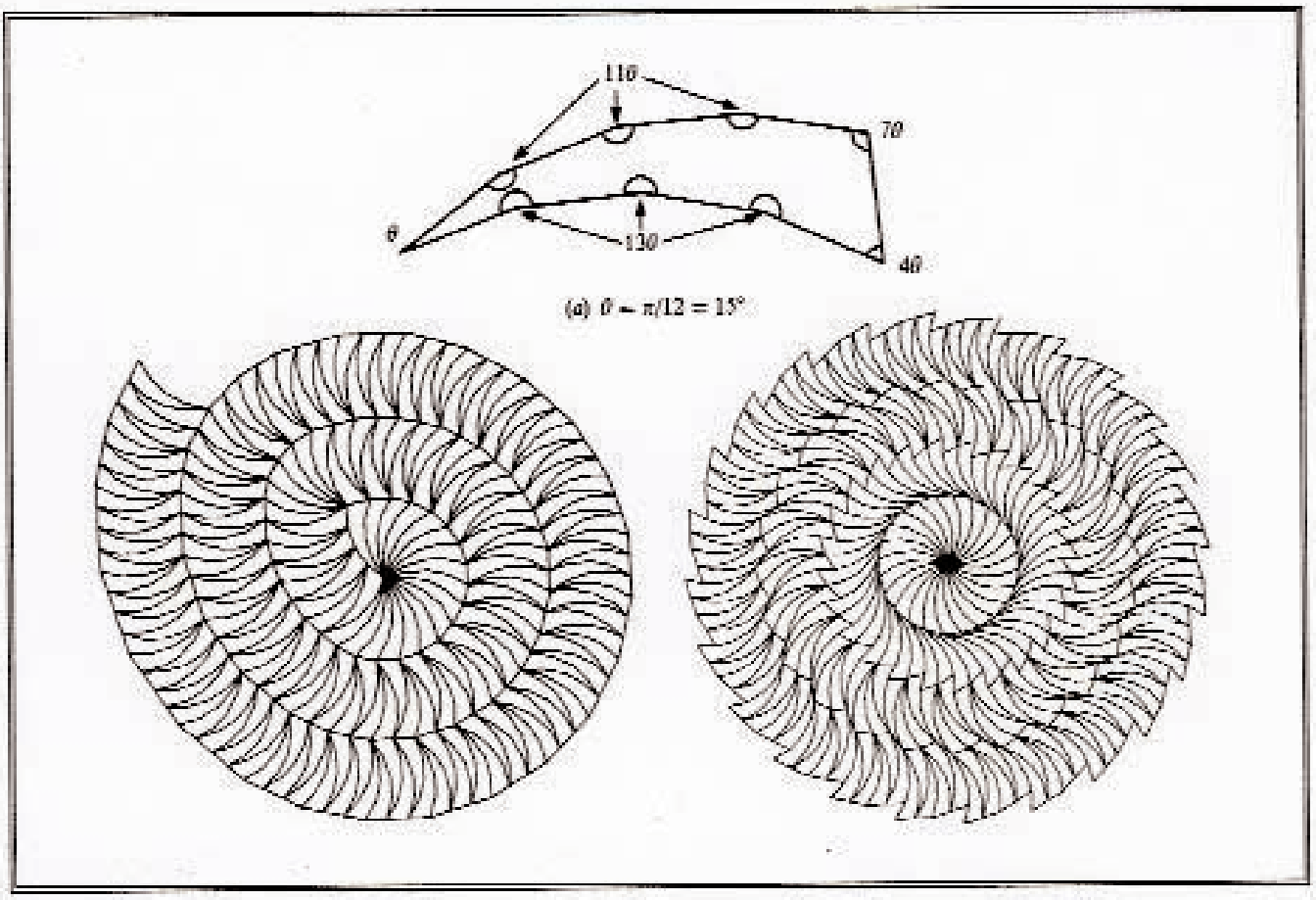


THE **REPTILES** ARE TILES THAT GROUP TOGETHER FORMING REPLICAS OF THEMSELVES.

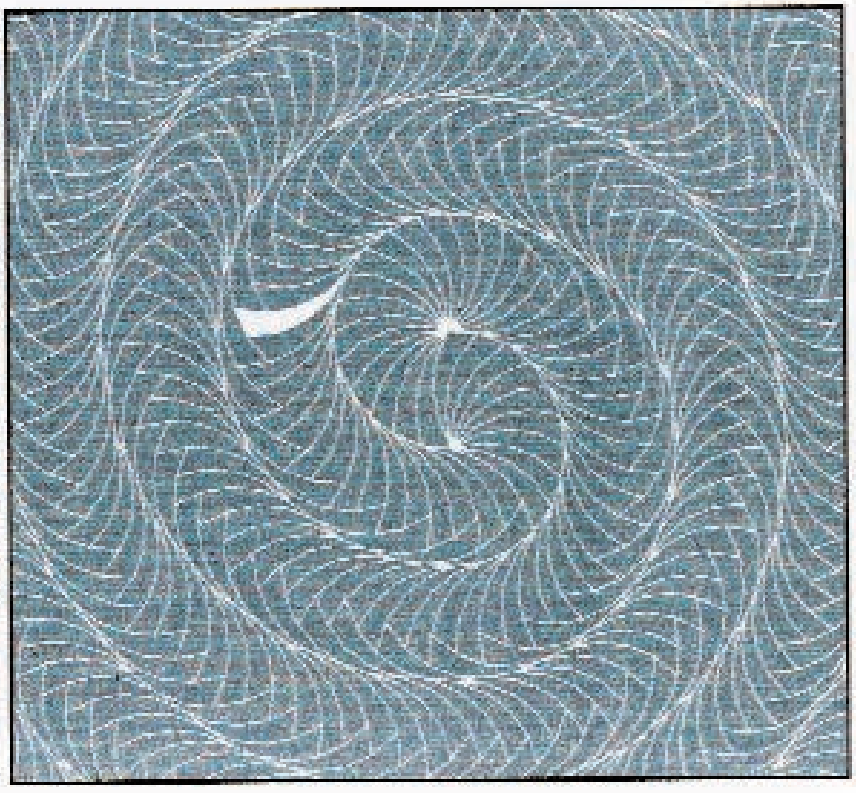


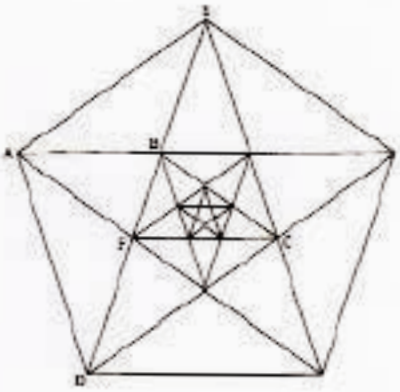
NON-PERIODIC OR APERIODIC TESSELLATION

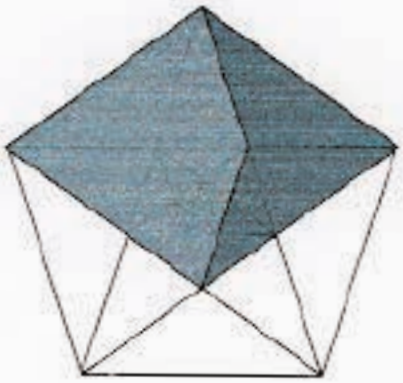
A NON-PERIODIC TESSELLATION
IS WHEN THE OVERALL DESIGN
FORMED BY ITS TILES
HAS A CENTRAL POINT
THAT EXPANDS
WITHOUT EVER REPEATING



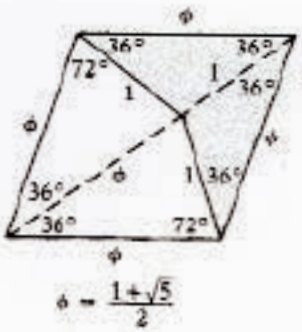
(a) $\theta = \pi/12 = 15°$

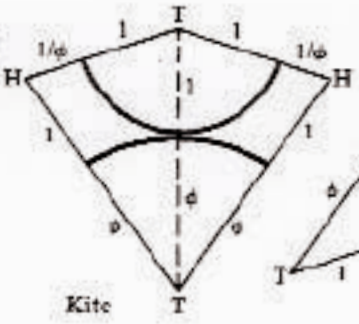


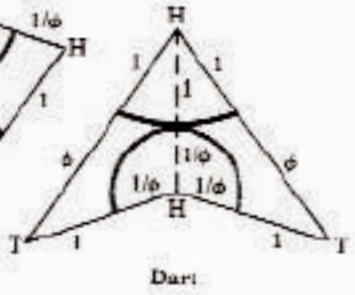




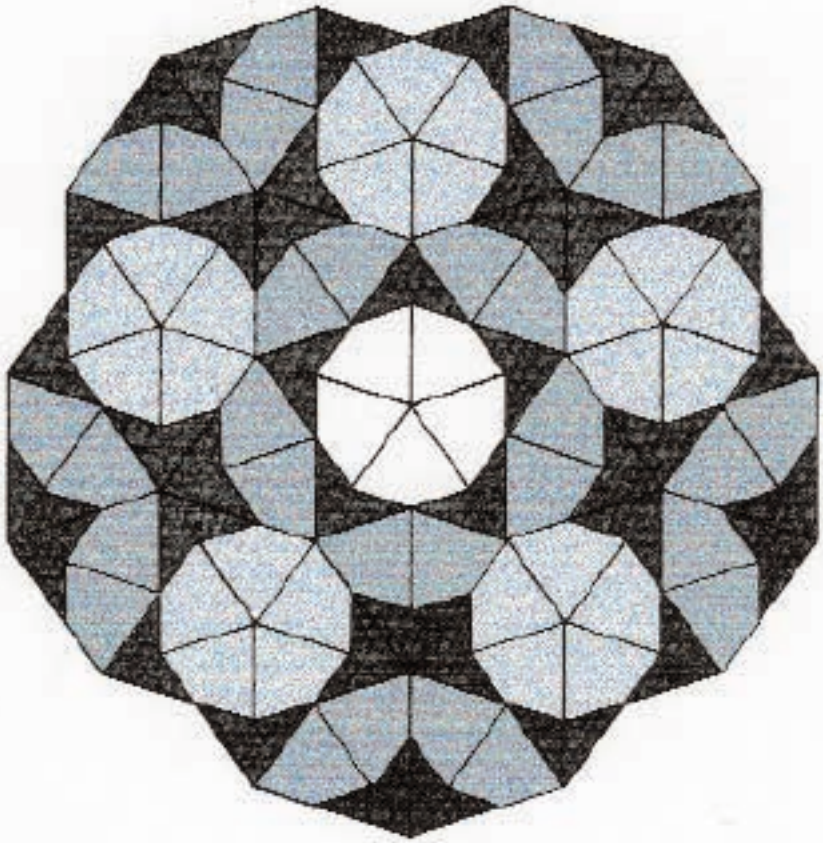
THE MOST FAMOUS NON-PERIODIC TILES ARE THE
"PENROSE TILES"



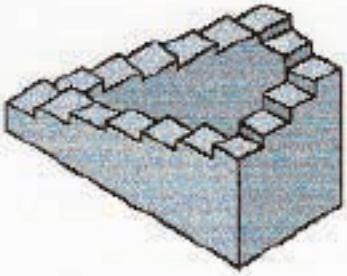




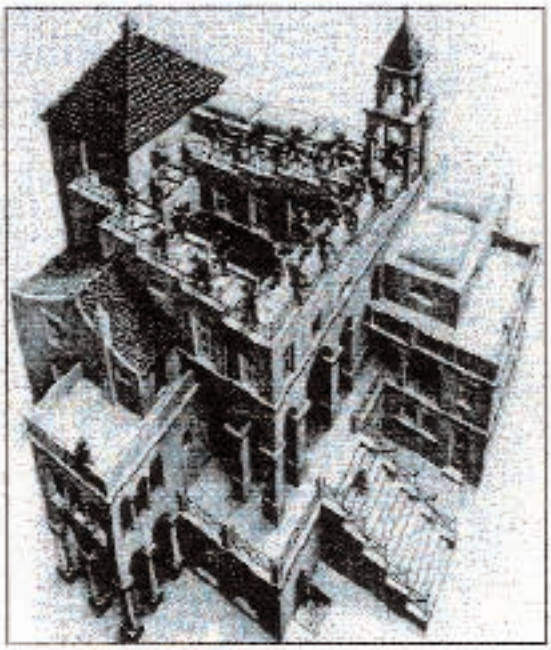
COMING FROM THE **PENTAGON**, DARTS AND KYTES HAVE
ITS MEASURES AND PROPORTIONS ALWAYS IN A "GOLDEN" RELATION WITH EACH OTHER,
TILING ONLY "**NON PERIODICALLY**".
THE QUANTITIES OF DARTS AND KYTES USED TO COVER A GIVEN AREA
ARE ALSO IN A GOLDEN PROPORTION,
SO, IF 1000 KYTES ARE USED, 618 DARTS WILL BE NEEDED.



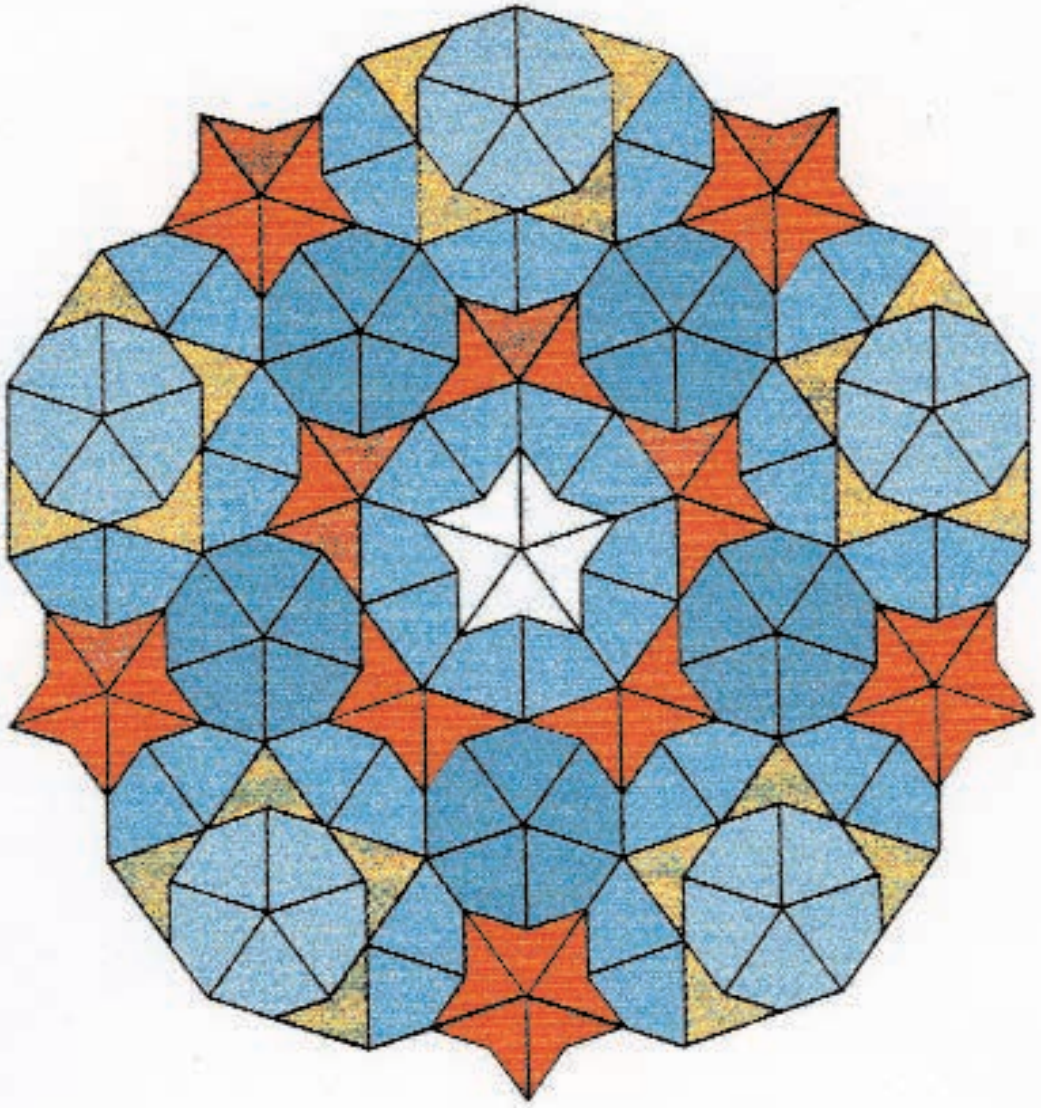
"The Infinite Sun Pattern"



These Poligons were discovered in 1974
by **Sir Roger Penrose**,
an English Mathematician and philosopher.
The same that, with his father, invented the
"endless Staircase"
used by M. C. Escher
on his print
"Ascending and Descending".

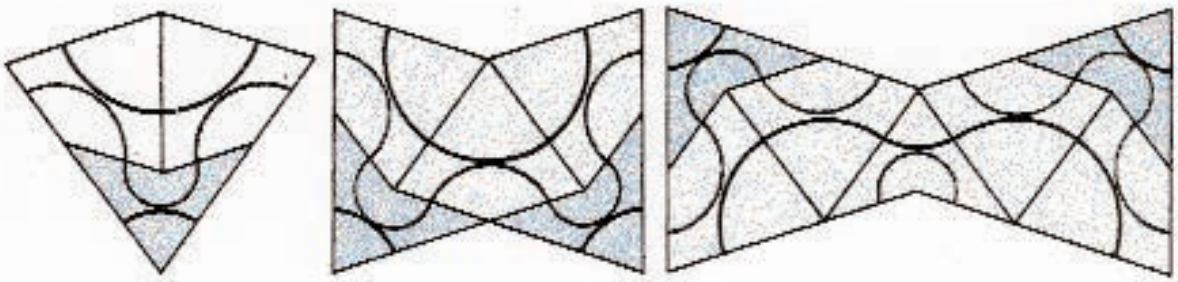


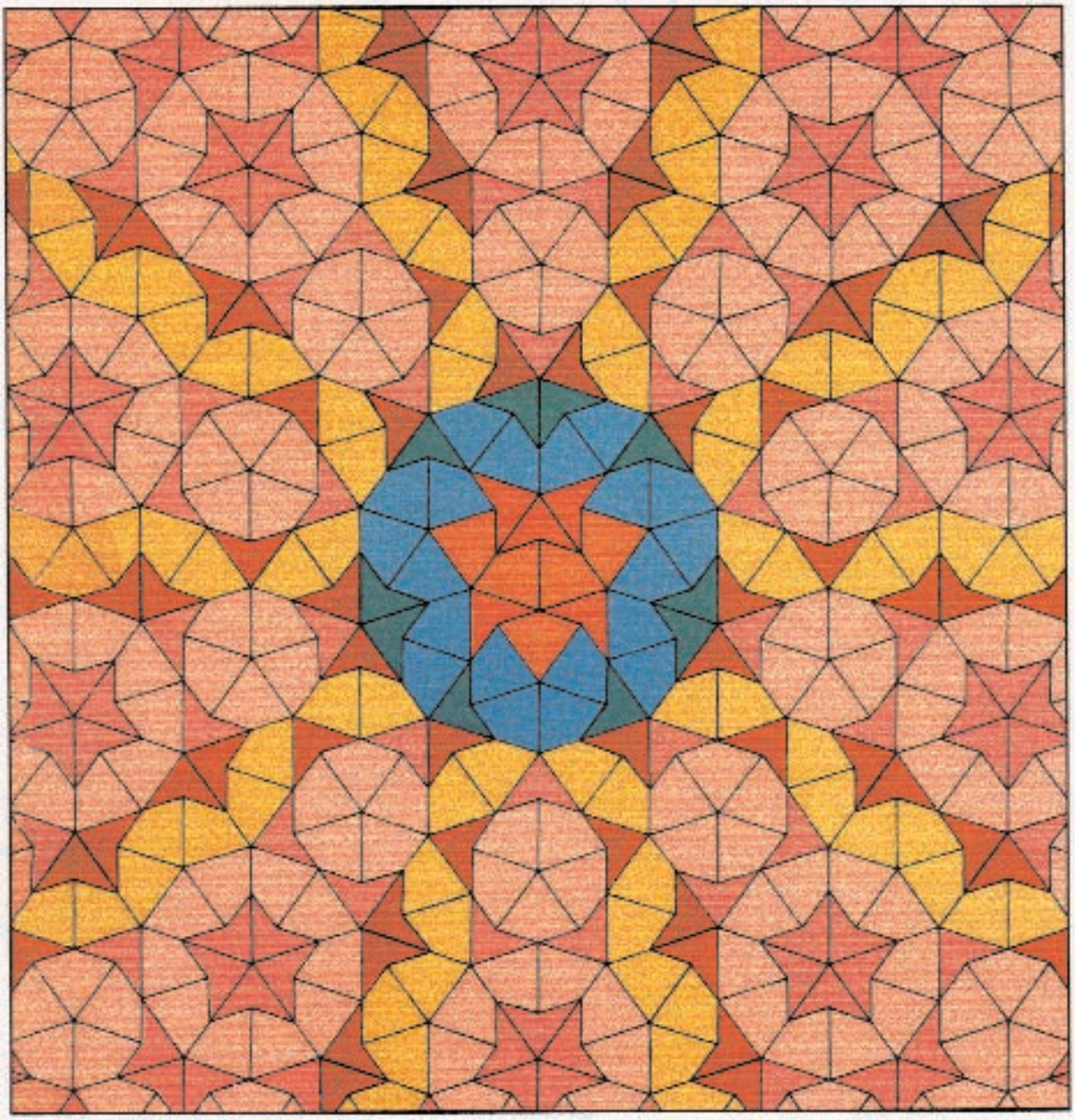
Ascending and Descending

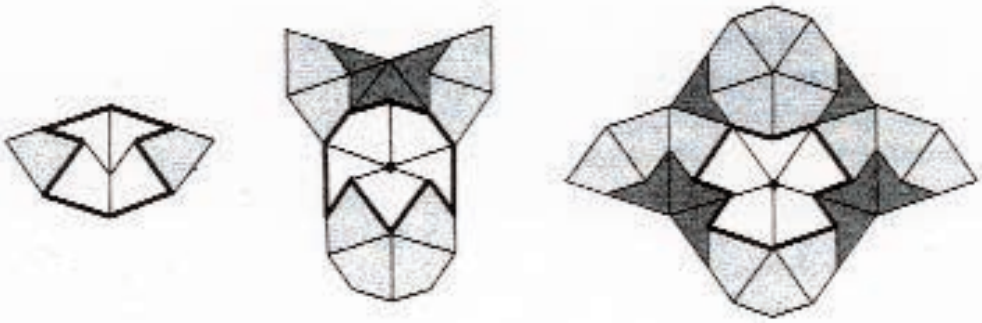


"The Infinite Star Pattern"

There are only SEVEN ways of assembling
DARTS and KYTES around a vertex
and the only ones with pentagonal symmetry are
"The Infinite Star Pattern"
and
"The Infinite Sun PAttern".

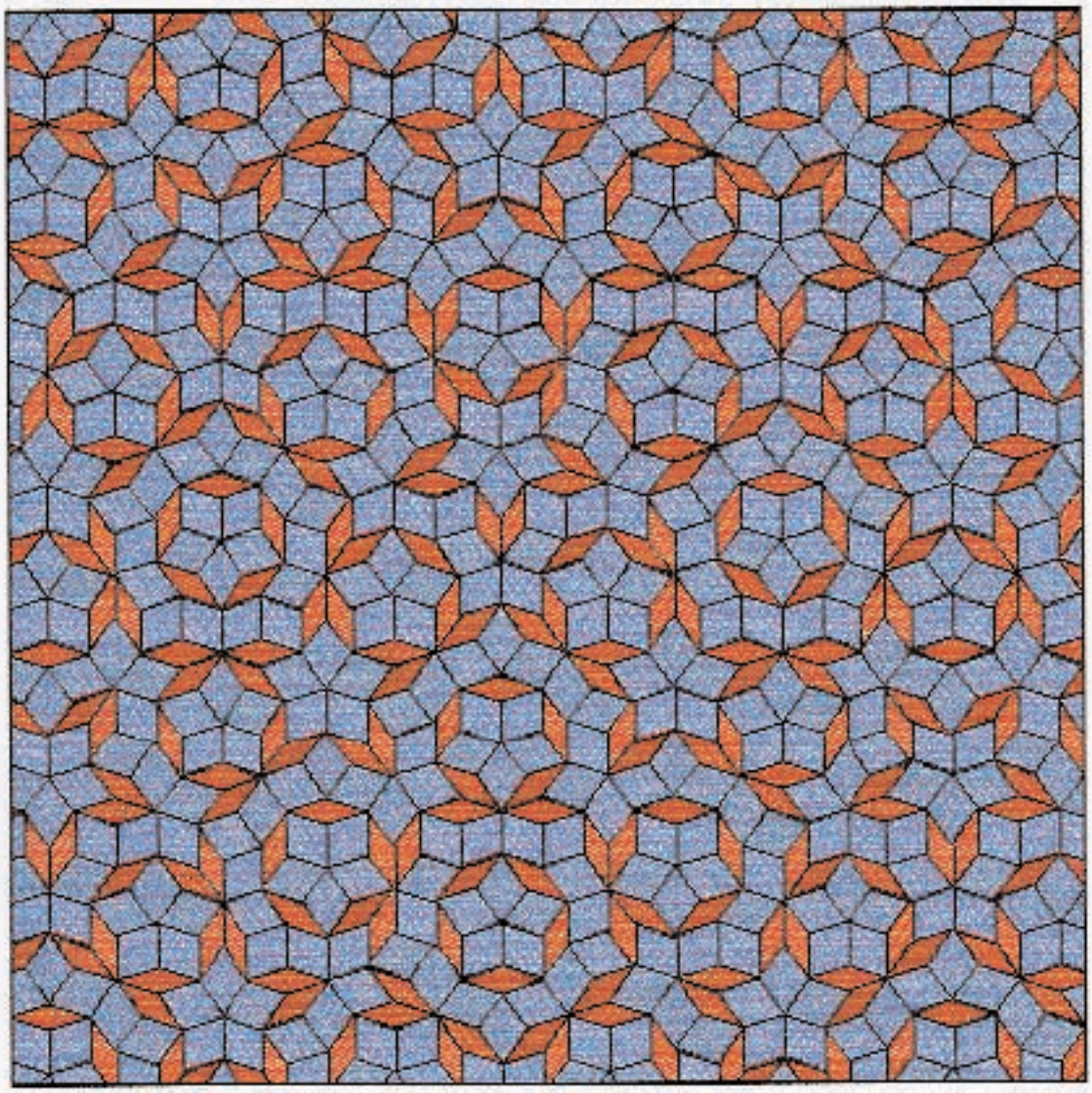


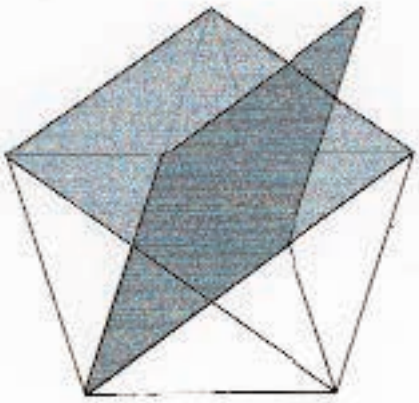




Deuce Jack Queen

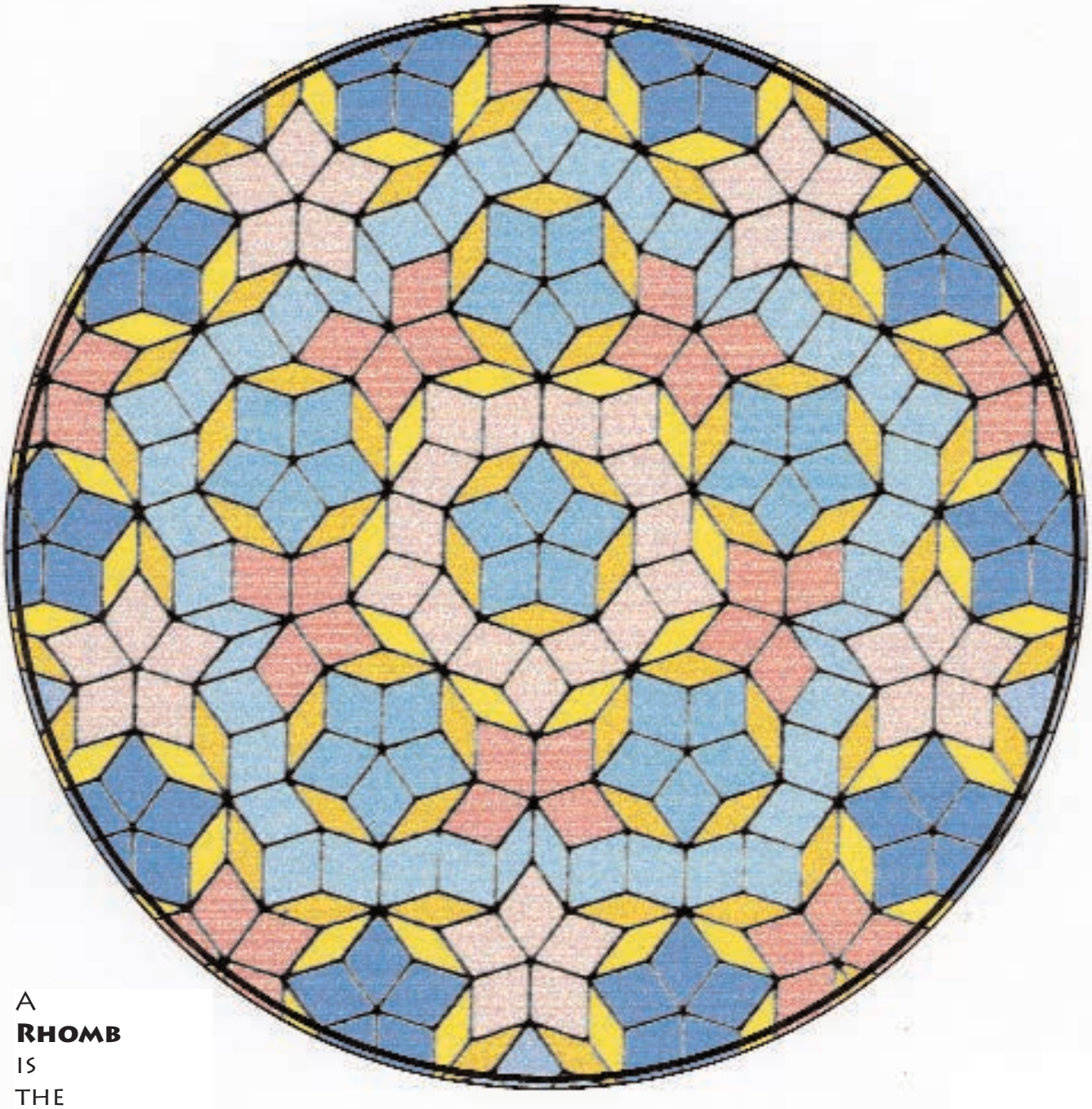
Another Variation of the Penrose Tile
The **LargeRhomb** and the **LongRhomb**



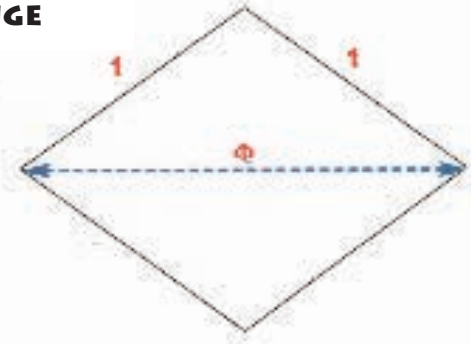


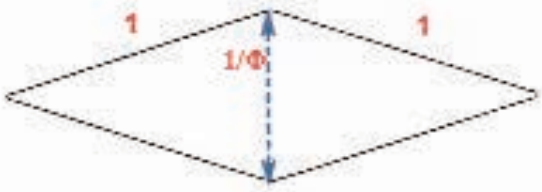
Long Rhomb

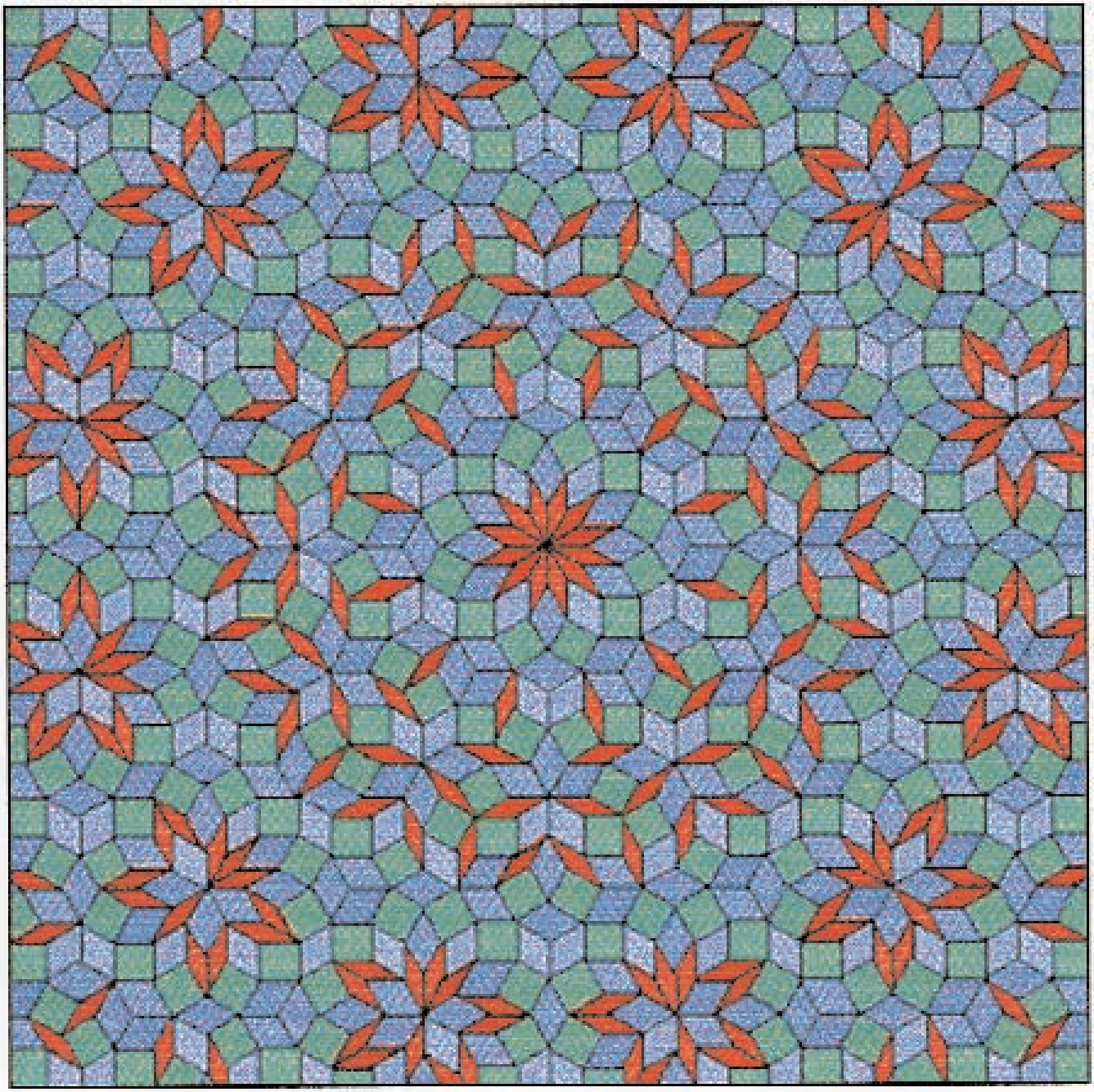
Large Rhomb



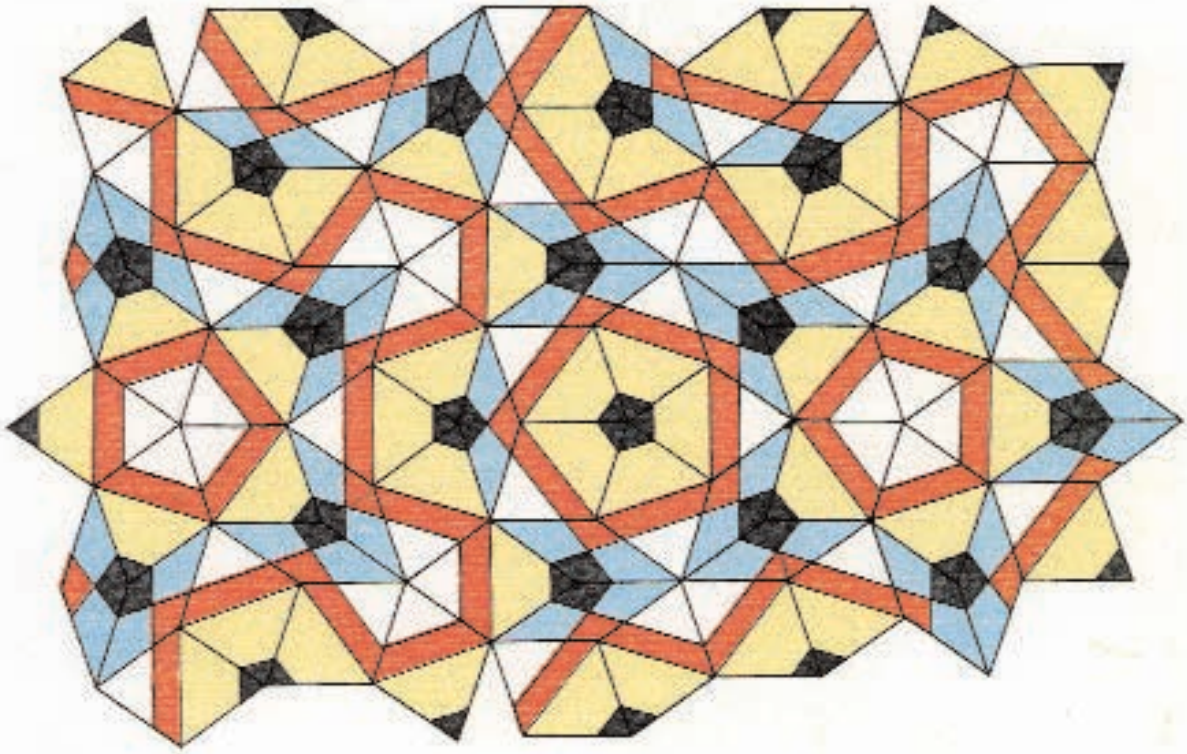
A
Rhomb
is
the
same
as
a
Lozenge



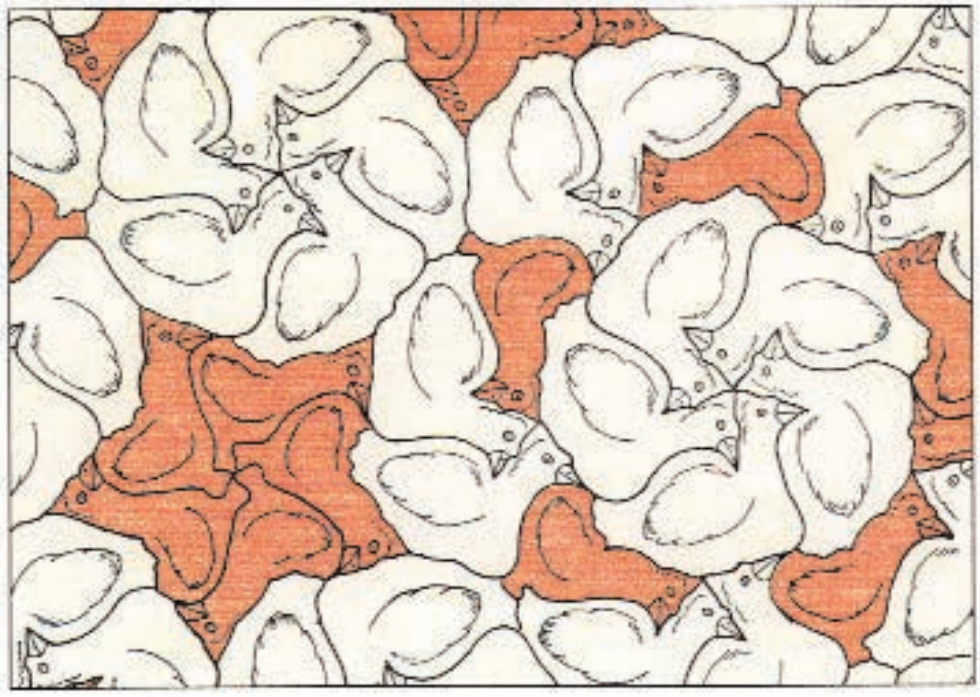




THE FIGURE ABOVE IS CALLED
"**QUASITILER**"
AND USES A SQUARE
TOGETHER WITH
THE LONG RHOMB
AND THE LARGE RHOMB
IN ORDER TO BUILD
THIS BEAUTIFUL
APERIODIC TESSELATION



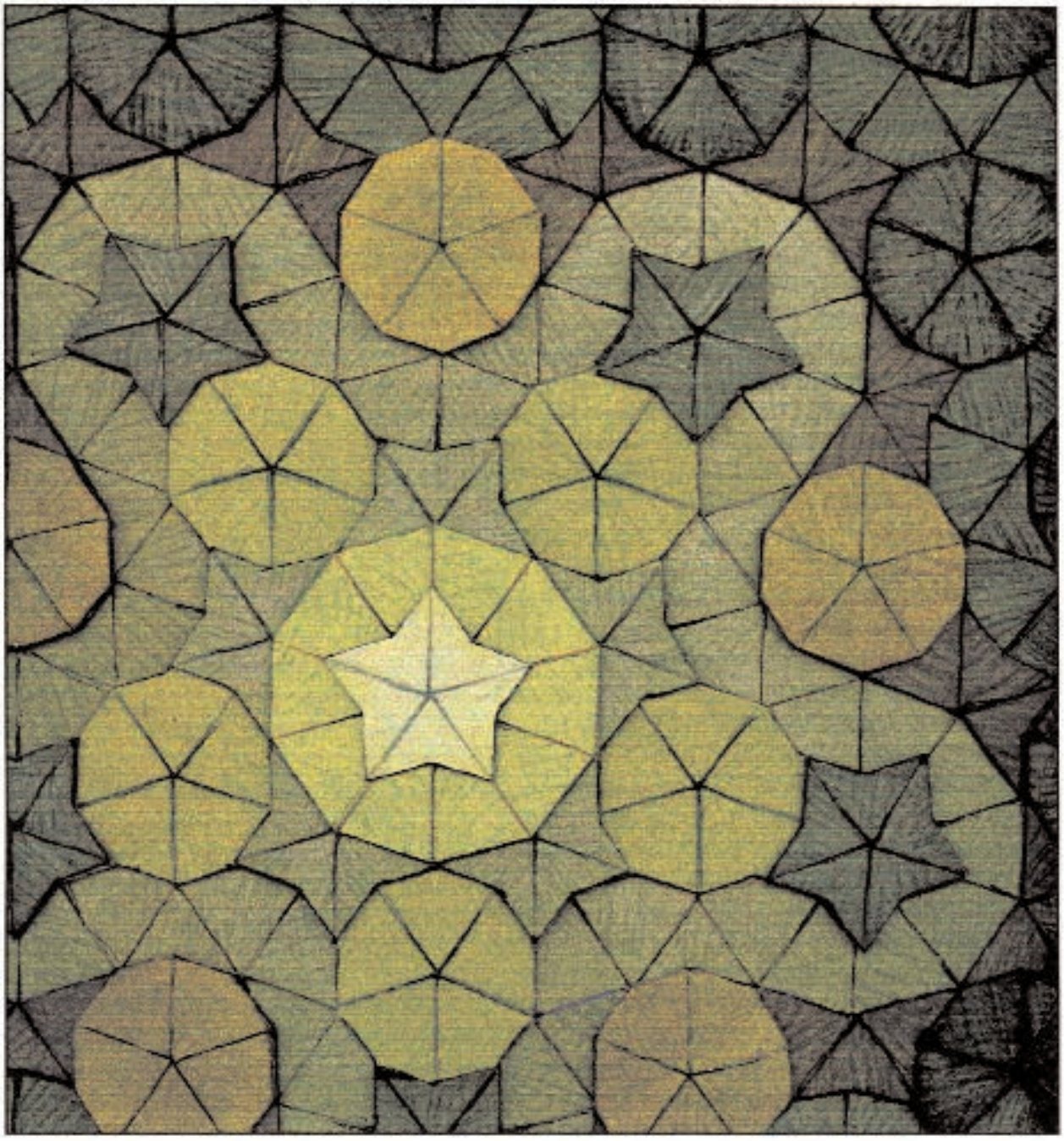
THE MARKS DRAWN IN THE FACES OF THESE
LONG RHOMBS AND LARGE RHOMBS
CONTRIBUTE IN THE FORMATION OF AN
EVER CHANGING GEOMETRIC DESIGN
THAT EXPANDS INFINITELY



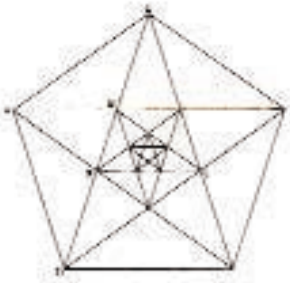
HEN 1

HEN 2

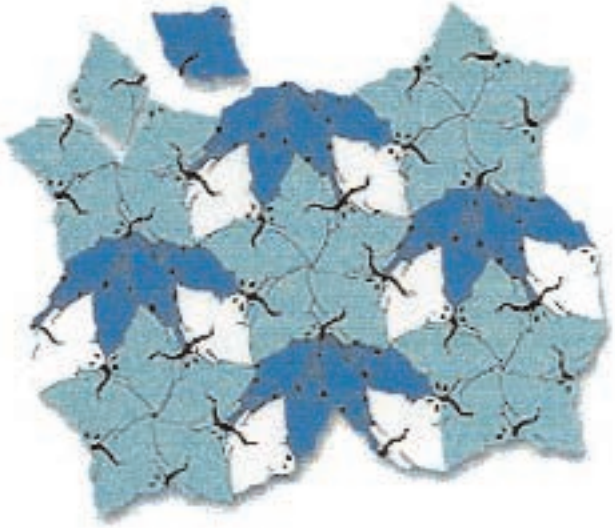
ADAPTATION OF THE PENROSE TILES DART AND KYTE
IN TWO "CHICKEN" SHAPES THAT TILE APERIODICALY

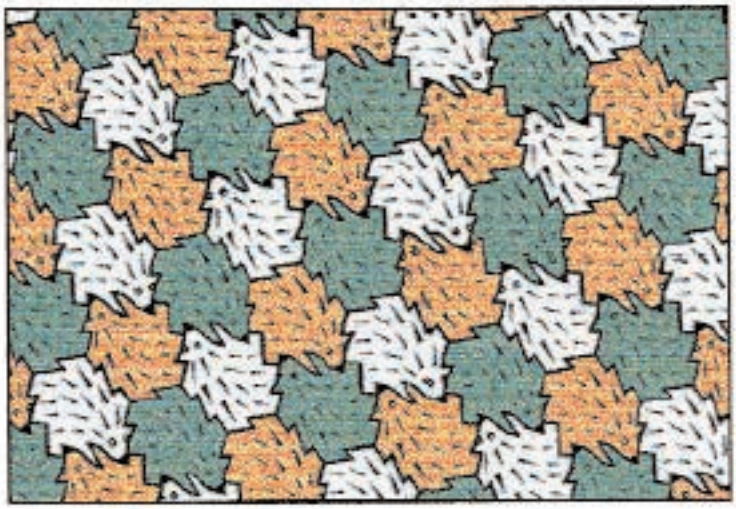


Project for a mural using multiple layers
of DARTS and KYTES
with different kinds of wood

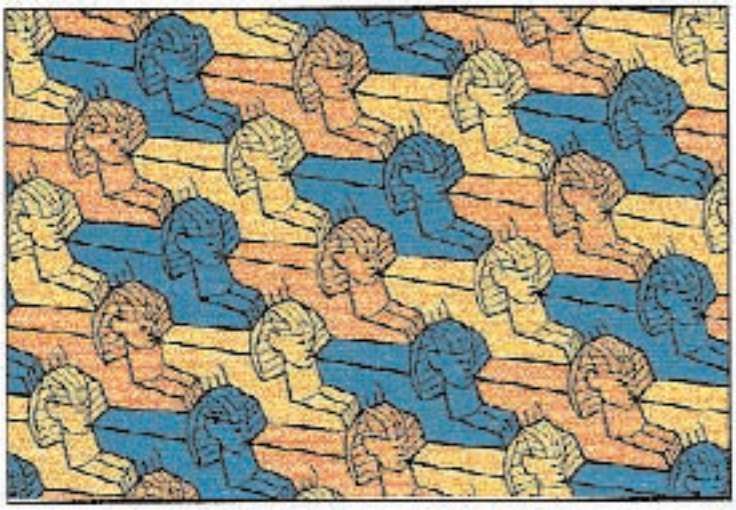


Some examples of
non periodic
tessellation





Some examples of
periodic tessellation



BIBLIOGRAPHY

"THE EYE BEGUILED"
Bruno Ernst
BENEDIKT TASCHEN

"THE EMPEROR'S NEW MIND"
Roger Penrose
OXFORD UNIVERSITY PRESS

"PENROSE TILES TO TRAPDOOR CIPHERS"
Martin Gardner
W. H. FREEMAN AND COMPANY

"TILINGS AND PATTERNS" AN INTRODUCTION
Branko Grunbaum & G. C. Shephard
W. H. FREEMAN AND COMPANY

"ESCHER ON ESCHER - EXPLORING THE INFINITE"
Maurits Cornelis Escher
HARRY N. ABRAMS, INC

e

"THE INTERNET"

TESSELLATIONS
http://www.cromp.com/tess/apc/page00.html

© 2014 Antonio Peticov

ANTONIO PETICOV
www.peticov.com.br
peticov@peticov.com.br
www.antoniopeticov.com.br